Signature of the Planck Scale

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Abstract

A fundamental spacetime scale in the universe leads to noncommutative spacetime and thence to a modified energy - momentum dispersion relation or equivalently to a modification of Lorentz symmetry as shown by the author and others. This latter consideration has also been used by some scholars though based on purely phenomenological models that have been suggested by the observation of Ultra High Energy Cosmic Rays. On the other hand a parallel development has been the proposal of a small but non zero photon mass by some scholars including the author, such a mass being within experimentally allowable limits. This too leads to a small violation of Lorentz symmetry observable in principle in very high energy gamma rays, as in fact is claimed. We show in this paper that the latter mechanism in fact follows from the former, thus unifying two apparently different approaches. We also examine this scenario in the context of Fermions and show some interesting results.

1 Introduction

Glashow and several others have considered a small modification of the velocity of light or of Lorentz symmetry based on hints from observation of Ultra High Energy cosmic Rays [1, 2, 3]. However the High Energy Gamma Ray data available at that time gave rather large limits for the variation of the speed of light or the mass of the photon, for example $\frac{\Delta c}{c} \sim 10^{-21}$ or m_γ, the photon mass less than $10^{-44} gms$ (Cf.ref.[4]). Independently the author

had come to a similar conclusion though from a purely theoretical point of view namely the existence of a fundamental spacetime scale, the Planck scale resulting in a noncommutative spacetime geometry [5, 6, 7, 8]. Such a conclusion about a violation of Lorentz symmetry also comes from a different and surprising angle as deduced by the author (and a few others) - that of a non zero photon mass proposed very early on by De Broglie, amongst others [9, 2, 10, 11, 12]. It may be mentioned that these latter values of the author are well within the tighter and improved experimental limits.

We will now argue that indeed the noncommutative spacetime approach, suggested by recent Quantum Gravity theories, leads directly to the same photon mass, thus unifying two approaches, that of a fundamental spacetime scale and that of the photon mass. We will also discuss the observational support for all this.

It should also be mentioned that in the author's work spacetime has an underpinning of Planck scale oscillators in a background Dark Energy - the ZPF, and this indeed had led in 1997 to the prediction of an accelerating universe with a small cosmological constant, besides several other consistent with observation results [13, 14, 6, 8]. Further it has been shown by the author and others that the Planck oscillator scale is a minimum - smaller scales are physically meaningless [2, 15, 16, 17].

2 The Modified Dispersion Relation

To see this in greater detail, we note that, given a minimum length l, we saw that the usual commutation relations get modified and now become

$$[x,p] = \hbar' = \hbar[1 + \left(\frac{l}{\hbar}\right)^2 p^2]\, etc \tag{1}$$

(Cf. also ref.[18]). (1) shows that effectively $\hbar$ is replaced by $\hbar'$. So, in units, $\hbar = 1 = c$,

$$E = [m^2 + p^2(1 + l^2p^2)^{-2}]^{\frac{1}{2}}$$

or, the energy-momentum relation leading to the Klein-Gordon Hamiltonian is given by,

$$E^2 = m^2 + p^2 - 2l^2p^4, \tag{2}$$

neglecting higher order terms. (It may be mentioned that some other authors as noted in the introduction have ad hoc taken a third power of p, and

so on. However we should remember that these were all phenomenological approaches.) Let us return to the Harmonic oscillators in the background Dark Energy. The theory of these Harmonic oscillators is well known (Cf. for example [19]). In this usual theory we have a super position of Harmonic oscillator solutions ($\hbar = 1 = c$)

$$f_k = e^{\imath (kx - \omega_k t)}$$

where we consider the one dimensional case, merely for simplicity and

$$\omega_k = \sqrt{k^2 + m^2} \tag{3}$$

Equation (3) is a reflection of the usual energy momentum relation

$$e^2 = p^2 + m^2$$

If now, we use instead the new dispersion relation (2) above we will get, as can be easily verified,

$$\omega_k^2 = k^2 + m^2 - 2l^2 k^4$$

This shows that there is a reduction in the energy given by

$$k_{eff}^2 = k^2 - 2l^2 k^4 \tag{4}$$

This is due to the appearance of a mass for the photon which would not be there in the usual theory with (3). Let us now estimate this photon mass. As can be seen from (4), we have,

$$m_\gamma = \left(\frac{\hbar}{c}\right) (k_{eff} - k), \tag{5}$$

where we have restored $\hbar$ and c. If we consider keV radiation as in the observations of Schaefer then we get for the photon mass a value $\sim 10^{-65} gms$. If on the other hand we consider TeV or GeV gamma rays, as are being observed then we can easily deduce from (5) that $m_\gamma \sim 10^{-62} gms$.
The important point is that latest observational estimates give an improved upper limit for the photon mass $\sim 10^{-57} gms$ [20, 21, 2]. Pleasingly our value is within this limit. It may be mentioned that exactly this photon mass was deduced by the author in the Planck oscillator - Dark Energy approach, quite different from the approach given above [9, 2]. Such a photon mass can also be deduced on purely thermodynamic considerations within the background

Dark Energy [22, 2]. Interestingly it was shown by Landsberg, using classical thermodynamic theory that the above deduced photon mass is the minimum allowable thermodynamic mass in the universe [23]. Exactly this mass was also proposed by Vigier and others [24] based on observational evidence.
It can be easily seen that this photon mass (or equivalently the above modified dispersion relation) leads to a dispersive velocity for the photon [24, 2]

$$v_\gamma = c \left[1 - \frac{m_\gamma^2 c^4}{h^2 \nu^2} \right]^{1/2} \tag{6}$$

Equation (6) shows the velocity dispersion with respect to frequency though this is a very subtle effect which can be observed in only Ultra High Energy Gamma Rays. Equation (6) in fact improves upon the limits of Schaefer and other authors, which were used by Glashow and other authors, as mentioned in the introduction. Moreover there have been claims that such a dispersive lag in the arrival of High Energy Gamma Rays has already been observed [25]. More recently Ellis and other authors have claimed such a dispersive lag in the time arrival of Gamma Rays from an event in the galaxy $mkn537$ [26].

3 Discussion

1. We may mention that the photon having a mass does not really contradict existing theory as pointed out by Deser [27].
2. That the photon has a mass can also be deduced directly from the background ZPF. Let us consider, following Wheeler a harmonic oscillator in its ground state. The probability amplitude is

$$\psi(x) = \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} e^{-(m\omega/2\hbar)x^2}$$

for displacement by the distance x from its position of classical equilibrium. So the oscillator fluctuates over an interval

$$\Delta x \sim (\hbar/m\omega)^{1/2}$$

The electromagnetic field for example is an infinite collection of independent oscillators, with amplitudes X_1, X_2 etc. The probability for the various oscillators to have amplitudes X_1, X_2 and so on is the product of individual

oscillator amplitudes:

$$\psi(X_1, X_2, \cdots) = exp[-(X_1^2 + X_2^2 + \cdots)]$$

wherein there would be a suitable normalization factor. This expression gives the probability amplitude ψ for a configuration $B(x, y, z)$ of the magnetic field that is described by the Fourier coefficients $X_1, X_2, \cdots$ or directly in terms of the magnetic field configuration itself by

$$\psi(B(x, y, z)) = Pexp\left(-\int\int \frac{\mathbf{B(x_1)} \cdot \mathbf{B(x_2)}}{16\pi^3 \hbar c r_{12}^2} d^3x_1 d^3x_2\right).$$

P being a normalization factor. Let us consider a configuration where the magnetic field or energy is everywhere zero except in a region of dimension l, where it is of the order of $\sim \Delta B$. The probability amplitude for this configuration would be proportional to

$$\exp\left[-\left((\Delta B)^2 l^4/\hbar c\right)\right]$$

So the energy of fluctuation in a volume of length l is given by finally [28, 29, 30]

$$\text{Energy} \sim \frac{\hbar c}{l} \tag{7}$$

We can see from the above equation (19) that this Energy in the background ZPF is a minimum where l is of the order of the radius of the universe, that is $\sim 10^{28} cm$. Substitution in (19) gives us back the above photon mass $m_\gamma \sim 10^{-65} gms$ [2]. As already pointed out this minimum mass agrees with the minimum mass allowable in the universe from usual thermodynamic theory, according to Landsberg.

3. Another way of looking at this would be that the background Dark Energy is a viscous medium with very small viscosity. This in fact has been shown to lead back to the above mass [22, 2]. Alternatively this can be seen in the following way [31]. The Maxwell equations in a vacuum with a non zero conductivity coefficient, can be shown to lead to a loss of energy of a photon during its propagation. This is because the dissipating mechanism leads to an extra term in the usual Maxwell's equations proportional to

$$\frac{\partial}{\partial t}\vec{E}$$

This has been shown to lead to the non zero photon mass.
So a non zero photon mass was obtained based on a background Dark Energy or ZPF (Cf. ref. [2]) at the Planck scale. On the other hand using the Planck scale as a minimum scale, it is known that spacetime geometry becomes noncommutative. This leads to a dispersion relation which is a modification of the usual Lorentz symmetry and equations like the Klein-Gordon and Dirac.
We have shown that, this alternative formulation leads to a photon with a mass which is exactly of the same order, thus unifying both the approaches. Finally not only is this photon mass well within the experimental limits, but also leads to observable results in the High Energy Gamma Ray spectrum. Hopefully NASA's GLAST satellite will throw further light on this.

4 The Modified Energy Momentum Formula for Fermions

For Fermions the analysis can be more detailed, in terms of Wilson lattices [32]. The free Hamiltonian now describes a collection of harmonic fermionic oscillators in momentum space. Assuming periodic boundary conditions in all three directions of a cube of dimension L^3, the allowed momentum components are

$$\mathbf{q} \equiv \left\{ q_k = \frac{2\pi}{L} v_k ; k = 1,2,3 \right\}, \quad 0 \leq v_k \leq L-1 \tag{8}$$

(8) finally leads to

$$E_{\mathbf{q}} = \pm \left(m^2 + \sum_{k=1}^{3} a^{-2} sin^2 q_k \right)^{1/2} \tag{9}$$

where $a = l$ is the length of the lattice, this being the desired result leading to

$$E^2 = p^2 e^2 + m^2 c^4 + \alpha l^2 p^4$$

((9) shows that α is positive.)

5 A Modified Dirac Equation

Once we consider a discrete spacetime structure, the energy momentum relation, as noted, gets modified [33, 32] and we have in units $c = 1 = \hbar$,

$$E^2 - p^2 - m^2 + l^2 p^4 = 0 \tag{10}$$

l being the Planck length. Let us now consider the Dirac equation

$$\{\gamma^\mu p_\mu - m\}\,\psi \equiv \{\gamma^\circ p^\circ + \Gamma\}\,\psi = 0 \tag{11}$$

If we include the extra effect shown in (10) we get

$$\left(\gamma^\circ p^\circ + \Gamma + \beta l p^2\right)\psi = 0 \tag{12}$$

β being a suitable matrix.
Multiplying (12) by the operator

$$\left(\gamma^\circ p^\circ - \Gamma - \beta l p^2\right)$$

on the left we get

$$p_0^2 - \left(\Gamma\Gamma + \{\Gamma\beta + \beta\Gamma\} + \beta^2 l^2 p^4\right\}\psi = 0 \tag{13}$$

If (13), as in the usual theory, has to represent (10), then we require that the matrix β satisfy

$$\Gamma\beta + \beta\Gamma = 0, \quad \beta^2 = 1 \tag{14}$$

It follows that,

$$\beta = \gamma^5 \tag{15}$$

Using (15) in (12), the modified Dirac equation finally becomes

$$\left\{\gamma^\circ p^\circ + \Gamma + \gamma^5 l p^2\right\}\psi = 0 \tag{16}$$

Owing to the fact that we have [34]

$$P\gamma^5 = -\gamma^5 P \tag{17}$$

It follows that the modified Dirac equation (16) is not invariant under reflections.
We can also see that due to the modified Dirac equation (16), there is no

additional effect on the anomalous gyromagnetic ratio. This is because, in the usual equation from which the magnetic moment is determined [35] viz.,

$$\frac{d\vec{S}}{dt} = -\frac{e}{\mu c}\vec{B} \times \vec{S},$$

where $\vec{S} = \hbar \sum /2$ is the electron spin operator, there is now an extra term

$$\left[\gamma^5, \sum\right] \tag{18}$$

However the expression (18) vanishes by the property of the Dirac matrices. We would like to comment on the modified Dirac equation (16). The modification is contained in the extra term $\gamma^5 lp^2$. This essentially is an extra mass that shows up that is the mass m of the fermion becomes $m + \Delta m$. However the curious feature is, that this extra term is, firstly independent of the mass m, and secondly as in (17) is not invariant under reflections.

If we now consider the case of a negligible mass, as in the two component neutrino theory [36], (16) shows that a supposedly massless particle acquires a mass, though this mass is not reflection invariant. We can thus see an explanation for the non zero neutrino mass.

Even if the fermion is massive, there is still the small correction Δm to its mass, though this again, according to (16) is not reflection invariant. It may be possible to detect this subtle effect in very high energy collision perhaps even in the context of LHC.

We finally observe that the term γlp^2 corresponds to an energy

$$E \sim 10^{21} eV$$

for an electron with a speed c, as can be easily calculated. More realistically,

$$E \sim l^2 m^2 \theta^2 c^2 \tag{19}$$

where θc is the particle's speed and m is its mass. For ultra relativistic protons with $\theta < 1$, (19) gives

$$E \sim \theta^2 \cdot 10^{27} eV$$

At this stage we comment on the above in a little detail, in the context of the violation of Lorentz symmetry as seen in (2), for example. It has been suspected that Lorentz symmetry is being violated from an observation of Ultra

High Energy Cosmic Rays (UHECR). In this case, given Lorentz symmetry there is the GZK cut off such that particles above an energy of about $10^{20}eV$ would not be able to travel cosmological distances and reach the earth (Cf. ref.[8, 14, 6, 37, 38, 39, 40] for details).
So detection of cosmic rays arriving at the Earth with energies above $10^{20}eV$ questions the presence of the GZK cutoff [41]. This cutoff determines the energy where the cosmic ray spectrum is expected to abruptly drop according to a power law in the energy. Cosmic rays with ultra high energies (above $\sim 5 \times 10^{19}eV$) lose energy through photoproduction of pions when traveling through the cosmic microwave background radiation (CMB). An event of $10^{20}eV$ has to be produced within $\sim 100Mpc$ unless there is non standard physics [42] and [43]. So these events are a mystery, the so called GZK puzzle. Does this mean that Lorentz symmetry is being violated?
It is suspected that some twenty contra events have already been detected, and phenomenological models of Lorentz symmetry violation have been constructed by Glashow, Coleman and others while this also follows from the author's fuzzy spacetime theory [44, 45, 46, 47, 48, 49]. The essential point here is that the energy momentum relativistic formula is modified leading to new effects.
What is very interesting is that already we are above the GZK threshold.
Finally, we point out that using the modified dispersion relation (2), but for Fermions, for a massless particle, $m = 0$, and identifying the extra term l^2p^4 as being due to a mass δm, we can easily deduce that

$$\delta m = \frac{\hbar}{cl} \,\mathrm{or}\, l = \frac{\hbar}{c\delta m}$$

This shows that l is the Compton wavelength for this mass δm or alternatively if l is the Compton wavelength, then we deduce the mass, now generated from the extra effect.

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